FRACTURE IN ROCK BASED ON THE EXTENSION STRAIN FAILURE CRITERION

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EXTENSION STRAIN

ROCK FAILURE CRITERIA (MOHR-COULOMB) DO NOT DESCRIBE WELL MINING EXCAVATIONS UNDER LOW STRESS CONDITIONS (NEAR SURFACE OF EXCAVATION)

- EXTENSION STRAIN FAILURE CRITERION

"FRACTURE OF ROCK WILL INITIATE WHEN THE EXTENSION STRAIN IN THE ROCK EXCEEDS A CRITICAL VALUE WHICH IS CHARACTERISTIC OF THAT ROCK TYPE"

$$e > e_c$$

- EXTENSION FRACTURES CAN FORM WHEN ALL THREE PRINCIPAL STRESSES ARE COMPRESSIVE

Rank	Name of mine	Depth	Location	Primary resource	Active / closed
1	Mponeng Gold Mine	4.0 km (2.5 mi)	South Africa	Gold[1]	Active
2	TauTona Mine	3.9 km (2.4 mi)	South Africa	Gold[1]	Active. Grouped under Mponeng [2]
3	Savuka Gold Mine	3.7 km (2.3 mi)	South Africa	Gold[1]	Closed 2017[3]
4	East Rand Mine	3.585 km (2.228 mi)[4]	South Africa	Gold	Closed 2008[2]
5	Driefontein Mine	3.420 km (2.125 mi)[5]	South Africa	Gold[1]	Active
6	Kusasalethu mine	3.388 km (2.105 mi)[6]	South Africa	Gold[1]	Active
7	Empire Mine	3.355 km (2.085 mi)	United States	Gold[7]	Closed 1956
8	Kloof mine	3.347 km (2.080 mi)[8]	South Africa	Gold, uranium	Active
9	Laronde Mine	3.260 km (2.026 mi)[9]	Canada	Gold, copper, silver, zinc[1]	Active
10	Kolar Gold Fields	3.217 km (1.999 mi)	India	Gold[1]	Closed 2001
11	Blyvooruitzicht mine	3.213 km (1.996 mi)[10]	South Africa	Gold, uranium	Active
12	Moab Khotsong mine	3.052 km (1.896 mi)[11]	South Africa	Gold, uranium[1]	Active
13	Kidd Mine	3.014 km (1.873 mi)[12]	Canada	Copper, zinc[1]	Active

CYLINDRICAL EXCAVATION

- AIMS: ILLUSTRATE AND MAKE QUANTITIVE PROPERTIES OF EXTENSION STRAIN

  LOOK FOR NEW PROPERTIES

- SIGN CONVENTION OF ROCK MECHANICS

  NORMAL STRESSES ARE POSITIVE IF COMPRESSIVE

$$\sigma_{ik} = -\tau_{ik} \qquad \varepsilon_{ik} = E_{ik}$$

- LATERAL STRESS COEFFICIENT OF PRE-MINING BACKGROUND

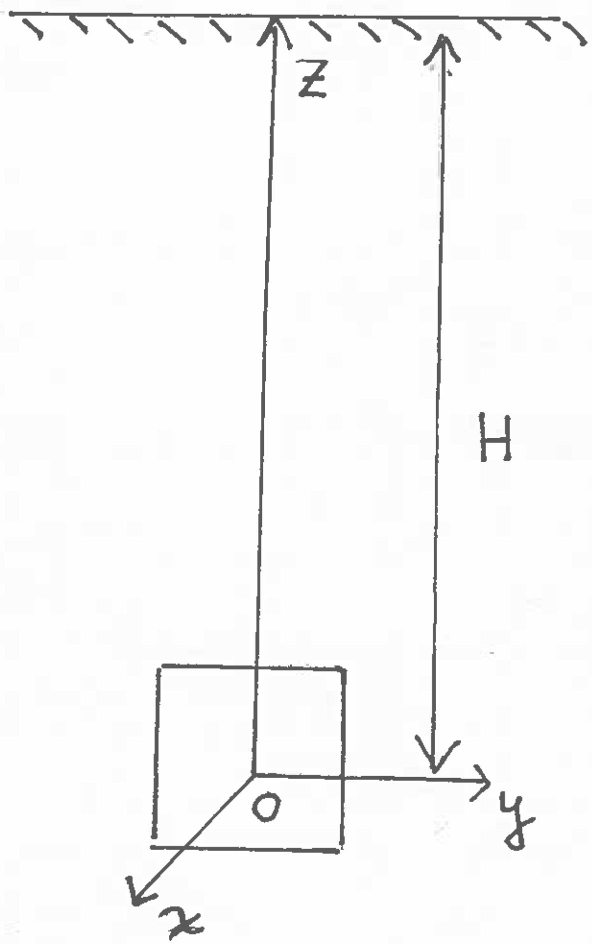
$$\sigma_{zz} = \rho g H$$

$$\sigma_{xx} = \sigma_{yy} = k\,\sigma_{zz} = k\,\rho g H$$

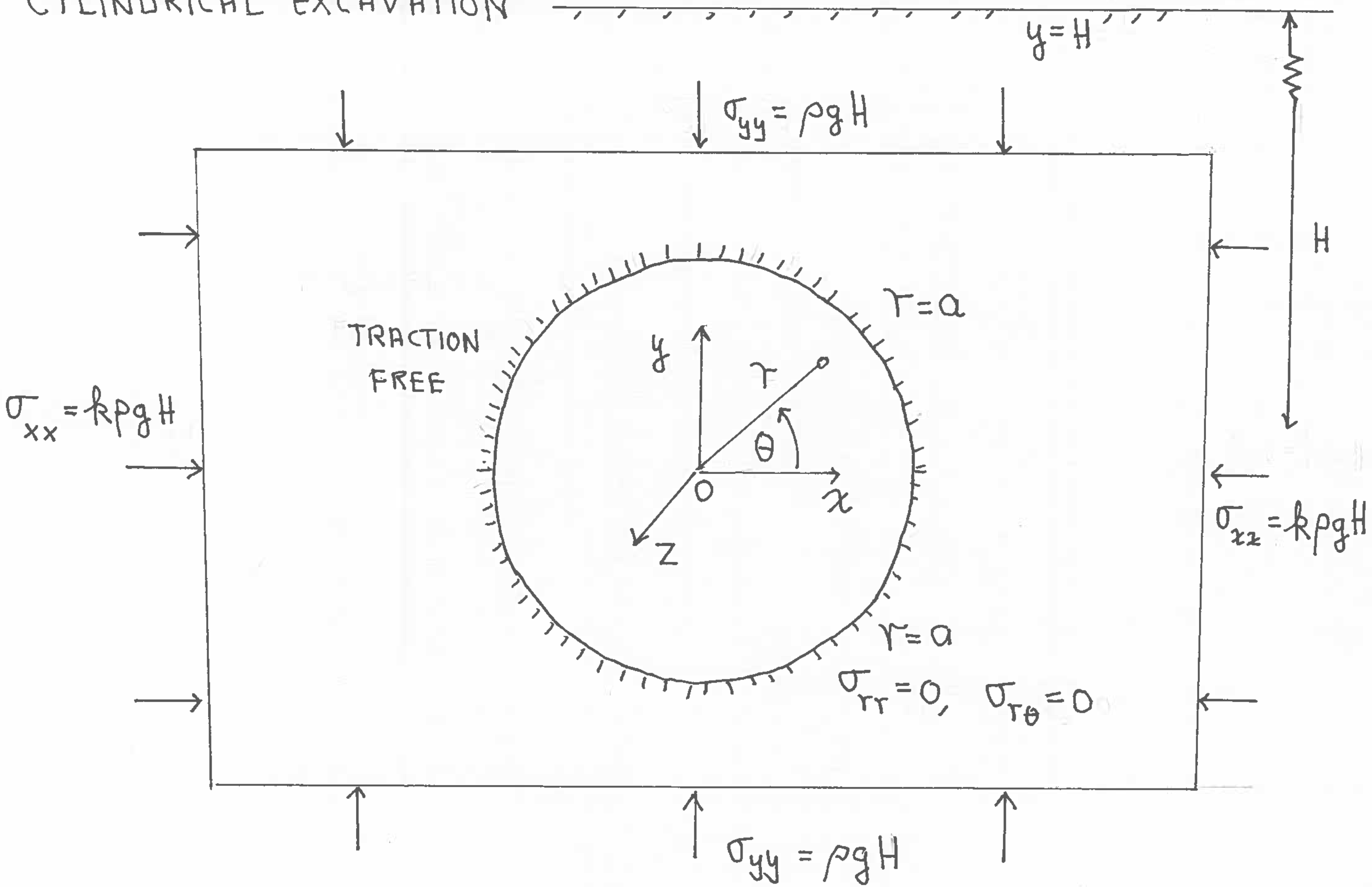
$$\sigma_{ij} = 0 \qquad i \neq j$$

$$0 \leq k \lesssim 4$$

HEIMES RULE $\quad k = 1$



CYLINDRICAL EXCAVATION



- LONG CYLINDRICAL EXCAVATION

  PLAIN STRAIN $\quad u_z = 0$

  $u_x = u_x(x,y), \quad u_y = u_y(x,y)$

- PRE-MINING BACKGROUND STRESS

  (BOUNDARY CONDITION AS $r \to \infty$)

  AIRY STRESS FUNCTION $U(x,y)$

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}, \quad \sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y}, \quad \sigma_{yy} = \frac{\partial^2 U}{\partial x^2}$$

BOUNDARY CONDITIONS

$$r \to \infty: \quad \sigma_{xx} = k\rho g H, \quad \sigma_{xy} = 0, \quad \sigma_{yy} = \rho g H$$

$$r \to \infty: \quad \frac{\partial^2 U}{\partial y^2} = k\rho g H, \qquad \frac{\partial^2 U}{\partial x \partial y} = 0, \qquad \frac{\partial^2 U}{\partial x^2} = \rho g H$$

$$r \to \infty: \quad U(x, y) = \frac{1}{2}\rho g H \left(x^2 + k y^2\right)$$

CYLINDRICAL POLAR COORDINATES

$$r \to \infty: \quad U(r, \theta) = \frac{1}{4}(1+k)\rho g H r^2 + \frac{1}{4}(1-k)\rho g H r^2 \cos 2\theta$$

- PROBLEM

SOLVE FOR $U(r,\theta)$ THE BIHARMONIC EQUATION

$$\nabla^4 U = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \right]^2 U = 0$$

SUBJECT TO BOUNDARY CONDITIONS

$r = a$: $\quad \sigma_{rr}(a,\theta) = 0, \quad 0 \leqslant \theta \leqslant 2\pi$

$r = a$: $\quad \sigma_{r\theta}(a,\theta) = 0, \quad 0 \leqslant \theta \leqslant 2\pi$

$r \to \infty$: $\quad U(r,\theta) = \frac{1}{4}(1+k)\rho g H r^2 + \frac{1}{4}(1-k)\rho g H r^2 \cos 2\theta$

WHERE

$$\sigma_{rr}(r,\theta) = \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2} + \frac{1}{r}\frac{\partial U}{\partial r}$$

$$\sigma_{r\theta}(r,\theta) = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial U}{\partial \theta}\right)$$

LOOK FOR SOLUTION OF FORM

$$U(r,\theta) = f_0(r) + f_2(r)\cos 2\theta$$

IT IS FOUND THAT $f_0(r)$ AND $f_2(r)$ SATISFY THE EULER EQUATIONS

$$r^4 \frac{d^4 f_0}{dr^4} + 2r^3 \frac{d^3 f_0}{dr^3} - r^2 \frac{d^2 f_0}{dr^2} + r \frac{df_0}{dr} = 0$$

$$r^4 \frac{d^4 f_2}{dr^4} + 2r^3 \frac{d^3 f_2}{dr^3} - 9r^2 \frac{d^2 f_2}{dr^2} + 9r \frac{df_2}{dr} = 0$$

$$\xi = \ln r$$

- CALCULATE STRESS COMPONENTS

$$\sigma_{rr}(r,\theta) = \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r}\frac{\partial u}{\partial r}$$

$$\sigma_{r\theta}(r,\theta) = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$$\sigma_{zz} = \nu\left(\sigma_{rr} + \sigma_{\theta\theta}\right)$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{\partial^2 u}{\partial r^2}$$

▧ EXTENSION STRAIN AT SIDE WALL $\theta = 0$

$\sigma_{r\theta} = 0$ PRINCIPAL STRESSES

INVESTIGATE RADIAL STRAIN

$$E_{rr} = \frac{\partial u_r}{\partial r} \qquad (u_r = \text{RADIAL DISPLACEMENT})$$

EXTENSION STRAIN AT HANGING WALL AND FLOOR

HANGING WALL (ROOF) $\theta = \frac{\pi}{2}$

FLOOR $\theta = \frac{3\pi}{2}$

AGAIN $\sigma_{r\theta} = 0$ PRINCIPAL STRESSES

INVESTIGATE RADIAL STRAIN

$$E_{rr} = \frac{\partial u_r}{\partial r}$$